Artificial Intelligence and Security Lab
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Deriving Smaller OA from Bigger Ones with GA

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WEPO 2021 – November 30, 2021

Orthogonal Arrays (OA)

- (N,k,s,t) **Orthogonal Array**: $N \times k$ matrix over $\{0,\cdots,s-1\}$ s.t. each t-uple occurs $\lambda = \frac{N}{s^t}$ times in each $N \times t$ submatrix.

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0

1	0	0
0	0	0
0	1	0
0	0	1
0	1	1
1	1	1
1	0	1
1	1	0

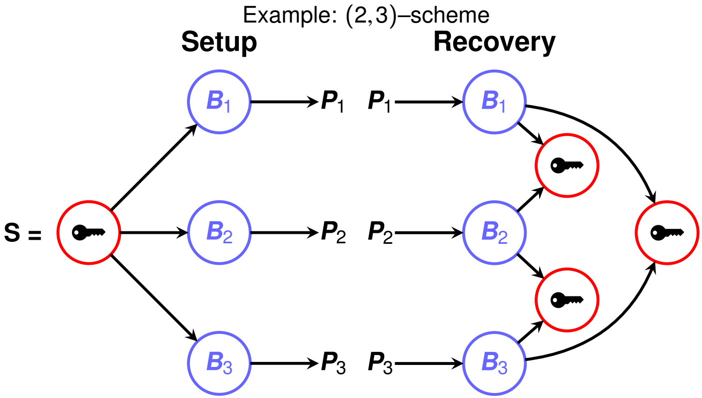
Example: OA $(8,4,2,3)$

$\Rightarrow$ Each 3-bit vector $(x_1,x_2,x_3) \in \{0,1\}^3$ appears once in the submatrix with columns 1, 3, 4

- Applications: designs of experiments, error-correcting codes, cryptography, ...

OA and Secret Sharing Schemes

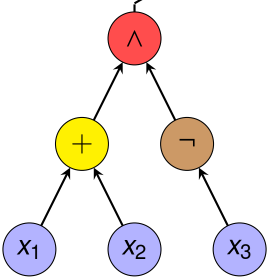
(t, n) **Threshold Scheme**: a **dealer** shares a **secret** S among n **players** so that at least t players are required to uniquely recover the secret S [B79, S79]



Remark: (t, n)–scheme $\Leftrightarrow$ OA $(N, n+1, s, t)$

Construction of OA by GA/GP [MPJL18]

- Focus on **Binary OA**: $s = 2$
- Column ⇔ truth table of a n-variable Boolean function
- For GP, the truth table is synthesized from the tree



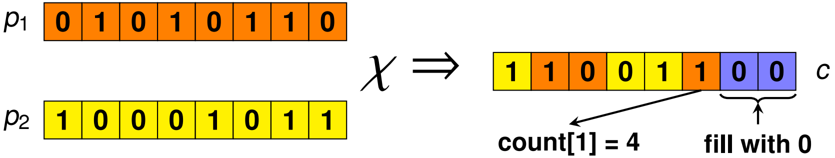
x_1	x_2	x_3	$f(x)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
0	1	1	1
1	0	0	1
1	1	0	1
1	1	0	0

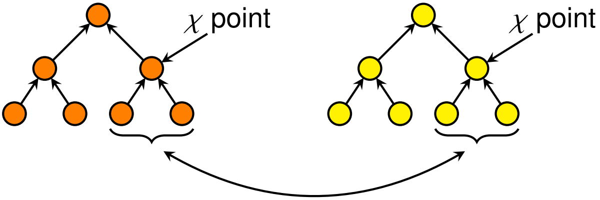
- Crossover and mutation are applied **column-wise**

Crossover Operators [MPJL18]

- **Remark**: Each column of an OA must be balanced
- **GA Crossover Idea:** Use *counters* to keep track of 0s and 1s
- Used for cryptographic Boolean functions [MCD98, ML15b]

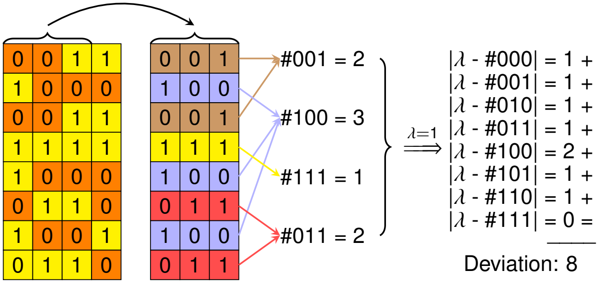


- **For GP**: Use standard subtree crossover



Fitness Function [MPJL18]

Idea: *minimize* in each $N \times t$ submatrix the number of occurrences of each t-uple deviating from λ

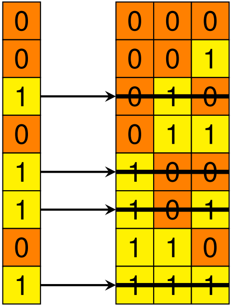


Fitness function: L^p distance between vector $(\lambda, \cdots, \lambda)$ and the vector of deviations for each submatrix

$$fit_p(A) = \sum_{S \text{ Submatrix}} \left(\sum_{x \in \{0,1\}^t} |\lambda - \#x|^p \right)^{\frac{1}{p}}$$

New problem: Removing rows from OA

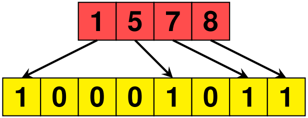
Problem statement: given an OA $(N, k, 2, t)$ with $\lambda = N/2^t$, find a smaller OA with $\lambda' < \lambda$ by removing $p = (\lambda - \lambda') \cdot 2^t$ rows



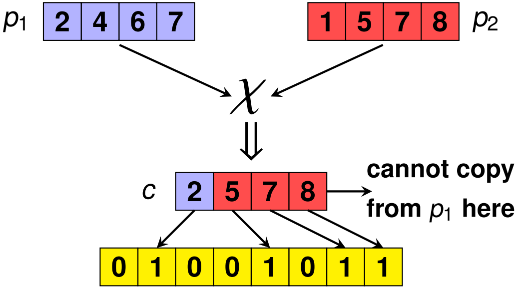
Question: How to choose the rows to remove?
Solution encoding for GA: N-bit string with p ones

Crossover Operator

Map of Ones Coding: Integer vector specifying the *positions of the* $p = (\lambda - \lambda') \cdot 2^t$ *ones* in the binary string



Idea: uniform crossover on the maps of ones, avoiding the insertion of duplicate positions in the child [MMT19, MMT20]



Experimental Setting

x_1	x_2	x_3	$x_1 \oplus x_2 \oplus x_3$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Basic test case: **zero-sum array** (ZS)

- First t cols.: all 2^t binary vectors in lexicographic order
- Col. $t+1$: XOR of the other cols.
- The whole matrix is an OA $(2^t, t+1, 2, t)$

OA/GA parameters:

- Initial OA: λ repetitions of ZS, with shuffled rows
- $t = 4$, $\lambda \in \{2,3,4\}$, $\lambda' \in \{1,2,3\}$
- Population size: 500 individuals, mutation probability: 0.2
- fitness budget: 100000 evaluations, runs: 30

Results

- **Main finding:** GA is able to converge with good success rate only on the smallest instance ($\lambda = 2$, $\lambda' = 1$)

λ, λ'	1	2	3
2	(24/30, 0.0)	–	–
3	(9/30, 7.07)	(4/30, 7.07)	–
4	(8/30, 7.07)	(0/30, 7.07)	(8/30, 7.07)

Table: Number of optimal solutions and median fitness

- Performances degrade quickly as soon as λ increases
- Best fitness follows a bi-modal distribution

Conclusions and Future Work

Recap:

- Genetic Algorithm for designing smaller Orthogonal Arrays starting from bigger ones
- The ones in the chromosome specify which rows to remove
- Experimental validation on a shuffled repetition of the zero-sum array

... Plenty of room for improvements!:

- Analyze the fitness landscape of this problem
- Systematic parameter tuning phase
- Use other crossover operators [MMT20, ML15b]
- Compare with other approaches on EA and combinatorial designs [SW92, MPJL17]
- Use other optimization methods (e.g., PSO [SUY06, ML15a])

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